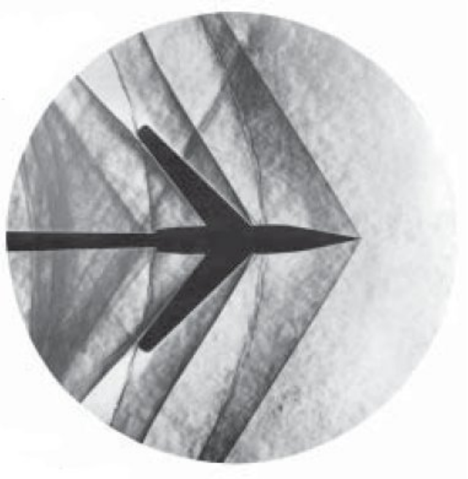
Mineral Physics I
Chapter 4. Equation of state
Section 8. Shock compression

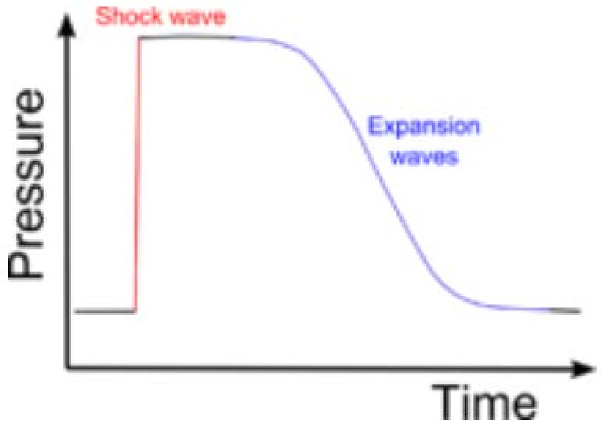
TOMOO KATSURA

BAYERISCHES GEOINSTITUT, UNIVERSITY OF BAYREUTH, GERMANY

Shock wave

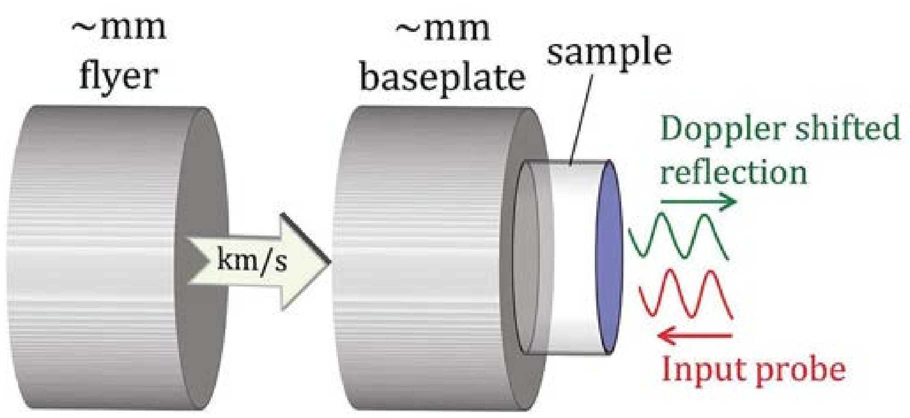
- ❑ Shock compression: an experiment to make a body under extremely high pressure using shock wave
- ❑ Shock wave:
  - ➢ A type of propagating disturbance that moves faster than the local speed of sound in the medium.
    - ✓ ex. associated with the passage of the ultrasonic aircraft and missile
  - ➢ Characterized by an abrupt, nearly discontinuous change in pressure, temperature and density of the medium

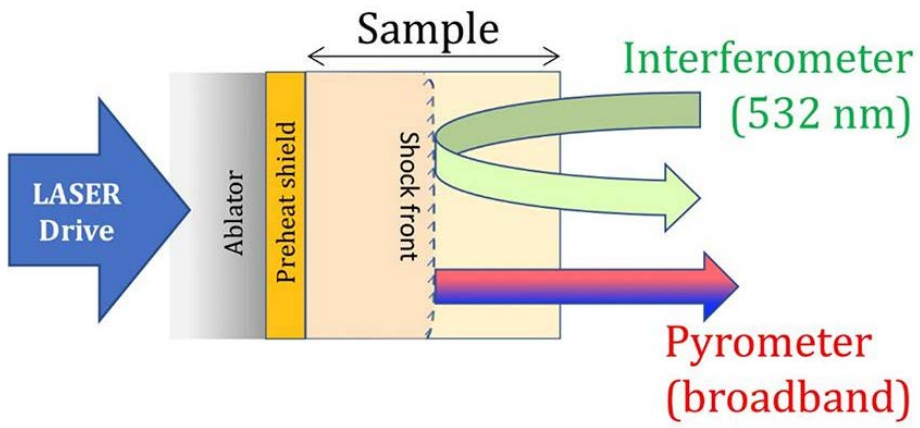




Shock compression

- ❑ Compress a solid or liquid by using a shock wave
  - ➢ Explosion
  - ➢ Gas-gun
    - ✓ Conventional technique
    - ✓ Collision of a high-velocity flyer accelerated by a gas-gun to a sample using a gun
  - ➢ Laser ablation
    - ✓ New technique
    - ✓ Instantaneous heating of the ablator by laser → sudden evaporation of the ablator → shock





Duffy & Smith (2019)

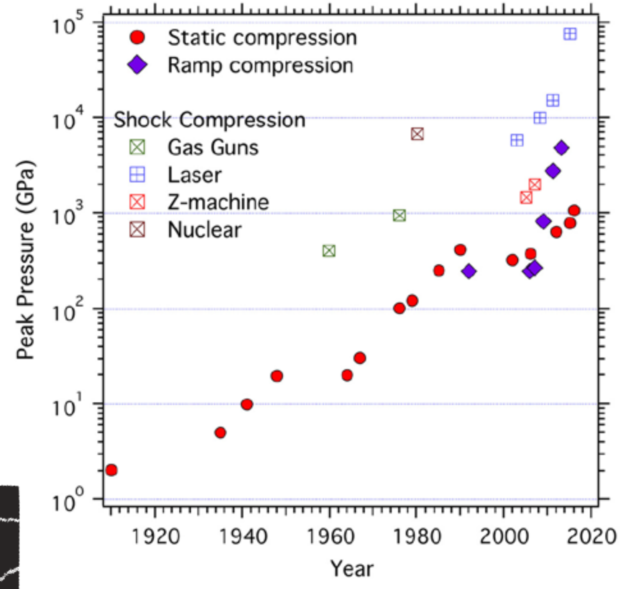
P, *T*, *t* conditions of shock compression

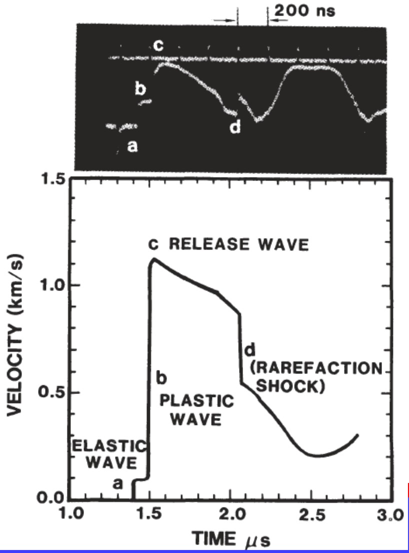
- *P* range: extremely high
  - Static compression using DAC
    - ✓ Can be 1 TPa, but mostly up to 200 GPa
  - Shock compression using a gas gun
    - ✓ Up to 1 TPa
  - Shock compression using the laser ablation
    - ✓ ~ 10 TPa or more

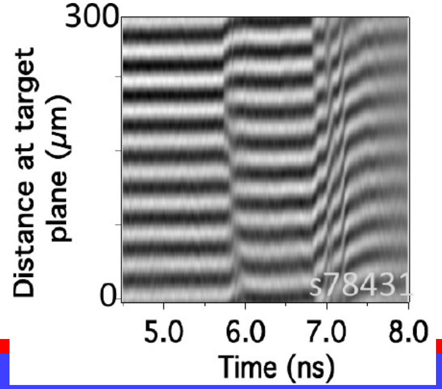
- *T* range: can be high according to the pressure and material
  - Can be 10,000 K in the TPa range
  - A large proportion of shock energy is used to increase the temperature in addition to the pressure

- Duration of high pressure: depending on the pressure and the size of the sample
  - a few μs ~ 1 ns

Duffy & Smith (2019)





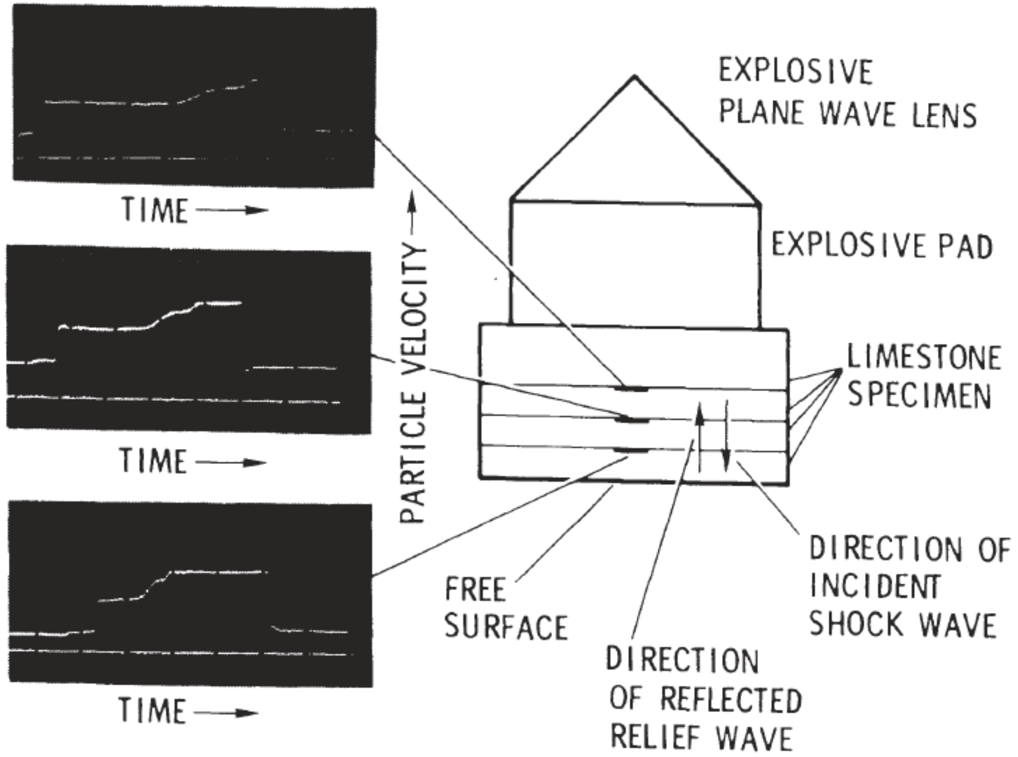


Advantage and disadvantage of shock experiment

- ❑ Advantages
  - ➢ Can generate extremely pressures
  - ➢ Can evaluate pressure without any pressure scale
  - ➢ Can evaluate density change
- ❑ Disadvantages
  - ➢ Very short (μs to ns) time scale
  - ➢ Temperature cannot be controlled independently from the pressure
    - ✓ Spontaneous temperature increase due to results of shock compression
  - ➢ Large errors in temperature estimation

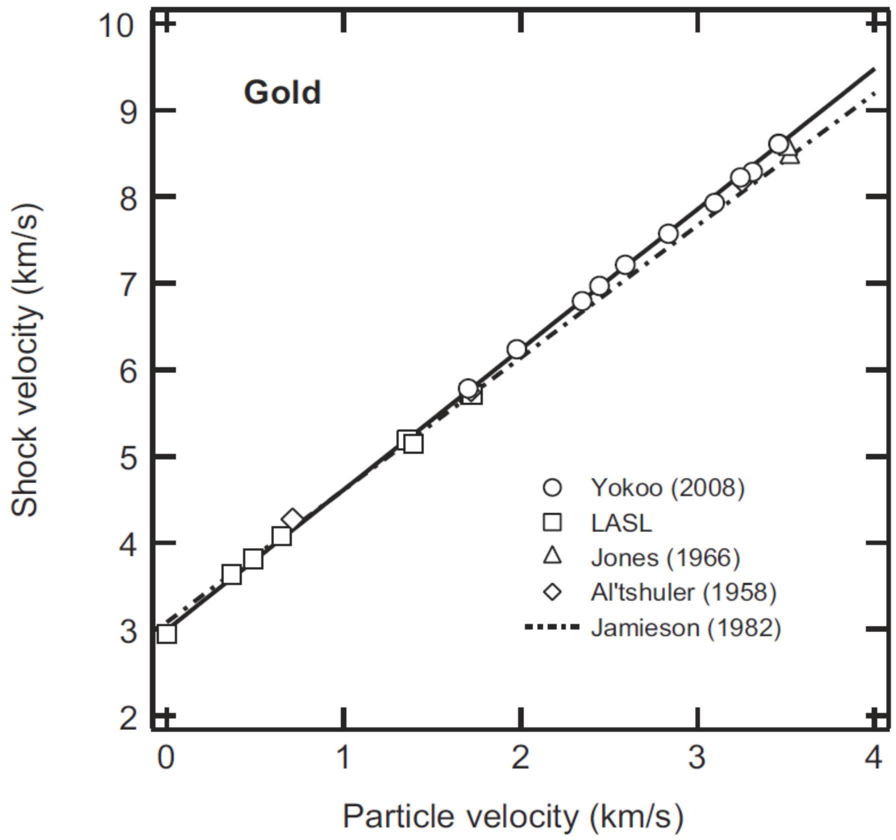
Gas-gun type shock compression

- ❑ This lecture: typically explain the gas-gun type shock compression experiment
  - ➢ Conventional, and therefore easy to understand
- ❑ Measured quantities
  - ➢ Velocity of the flyer (projectile)
    - ✓ "particle velocity"
    - ✓ Regarded as the velocity of motion of atoms or particles composing the material
  - ➢ Shock velocity



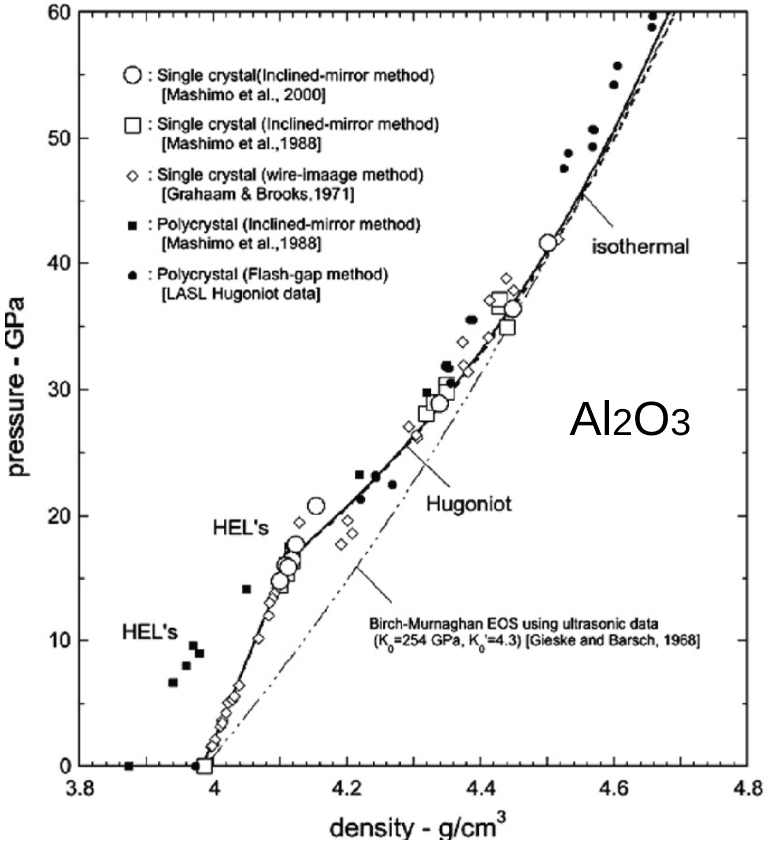
An example of shock compression data

- Example: gold
  - $U_s = 2.995 + 1.653u_p - 0.013u_p^2$ (4.8.1)
    - U_s: shock velocity
    - u_p: particle velocity
      - Los Alamos Shock Hugoniot Data, edited by S. P. Marsh, University of California Press, Berkeley, 1979; A. H. Jones *et al.*, *J. Appl. Phys.* 37, 3493, 1966; L. V. Al'tshuler *et al.*, *J. Appl. Mech. Tech. Phys.* 22, 145, 1981; M. Yokoo et al., *Appl. Phys. Lett.* 92, 051901, 2008.



Hugoniot elastic limit

- ❑ Solid: having significant to the shear
  - ➢ resist to shock wave up to some conditions
- ❑ Hugoniot elastic limit (HEL)
  - ➢ The matter behaves as a liquid above the HEL
  - ➢ The data below the HEL cannot be used to obtain an equation of state
  - ➢ Examples:
    - ✓ Al_2O_3: about 20 GPa (very hard)
    - ✓ Au: almost none (very soft)

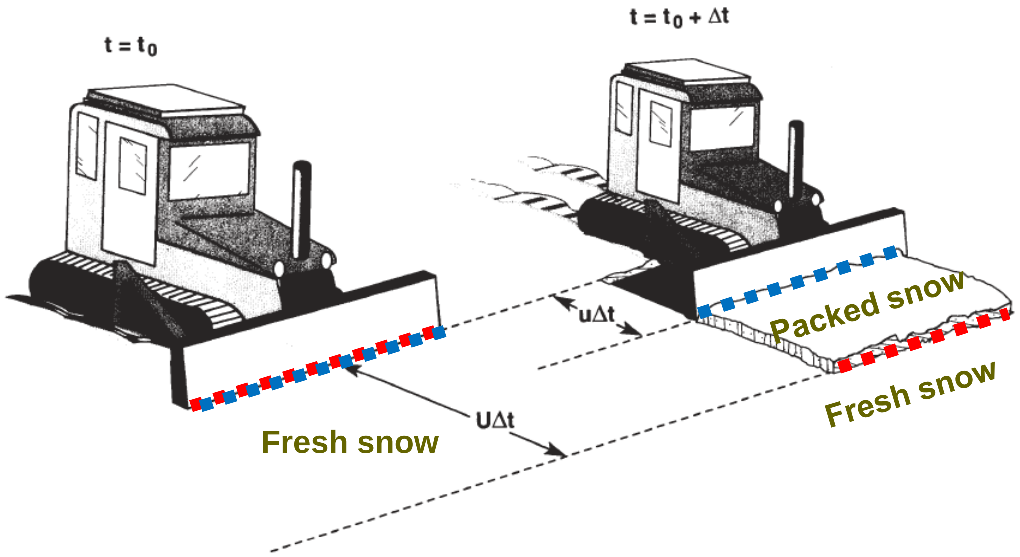


Al_2O_3

Physical image of shock wave: analogy of a moving slowplow

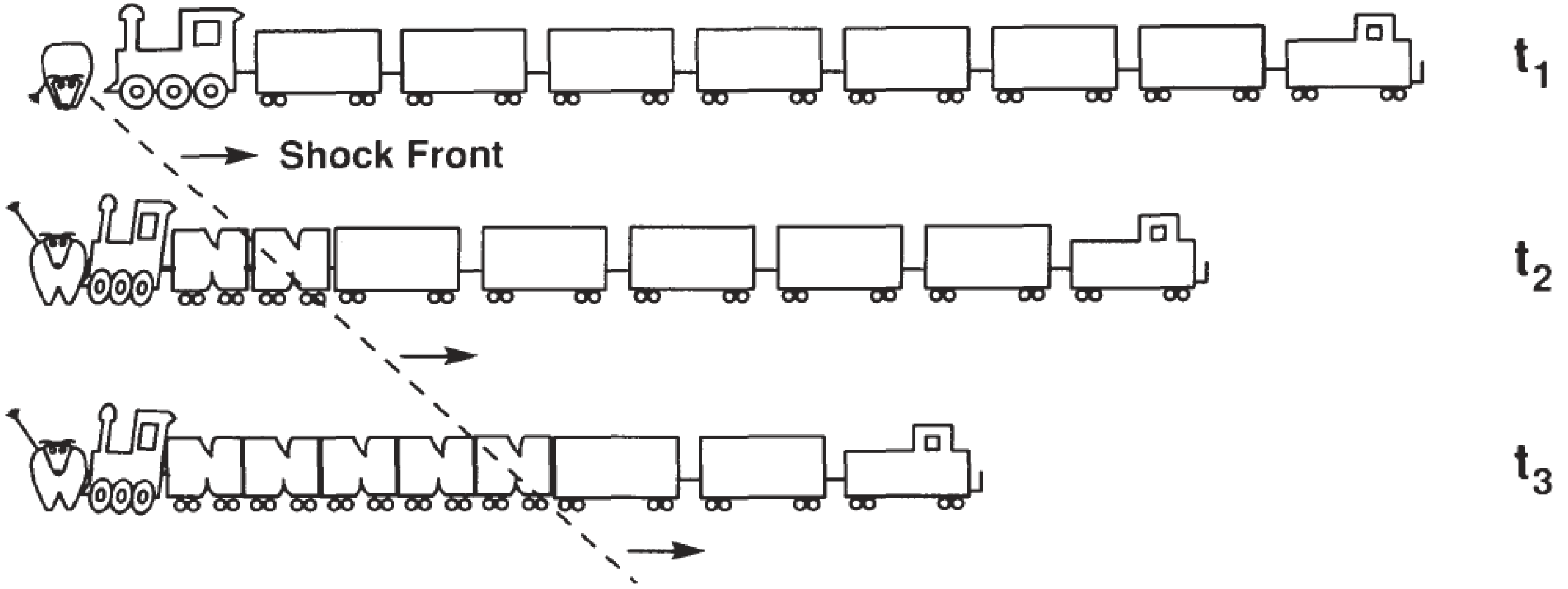
- ❑ Shock wave: Ubiquitous results of matters moving at velocities faster than the speeds at which adjacent material can move

- ❑ Example: the flow of snow in front of a moving snowplow
  - ➢ Movement of plow into fresh loose snow → formation of packed snow
    - ✓ The interface of fresh and packed snows moves faster than the plow



- ➢ Blue line: the front of plow
- ➢ Red line: the interface of the fresh and packed snows

Physical image of shock wave: analogy of a moving train



Assuming that the bull is so strong that it stays the collision point.

- ❑ A bull is struck by a moving train
  - ➢ Shock wave: a moving discontinuity that separates undisturbed boxcars from crushed ones
  - ➢ Moving from left to right away from the point of collision

Particle and shock velocities

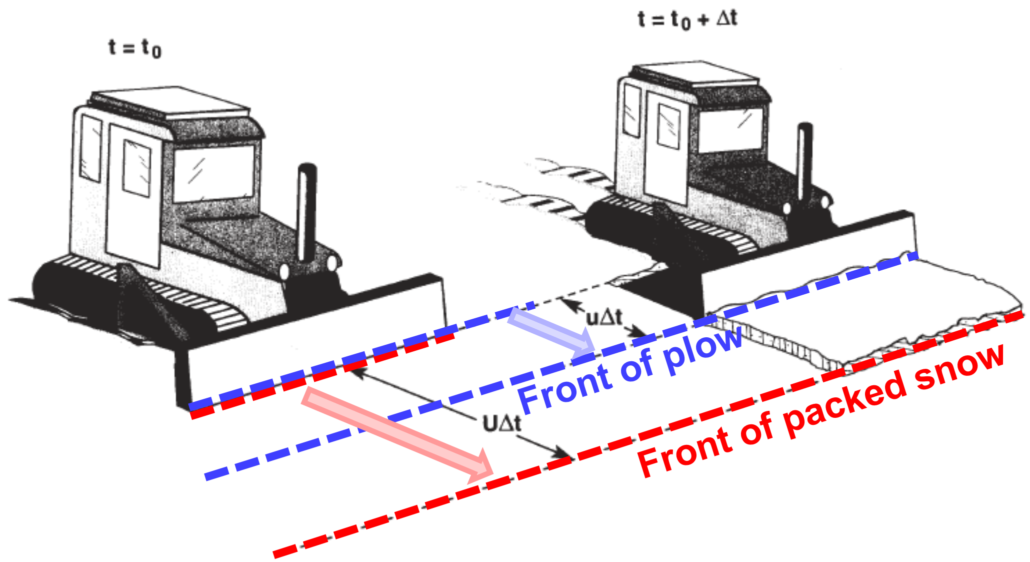
- Particle velocity: the velocity that a given element in the material acquires as a result of the shock wave passing over the element
  - We approximate that the elements have no velocity before the shock wave arrives.
- Shock velocity: the velocity with which the disturbance moves through the body
- Shock velocity >> particle velocity
  - Ex. Debris such as dust or paper blown by a gust of wind
    - ✓ Shock velocity → the wind velocity
    - ✓ Particle velocity → the velocity of debris movement
    - ✓ The wind velocity >> the debris velocity

Equation for shock wave

- The fundamental points:
  - Mass, momentum, and energy are conserved across the shock discontinuity
  - → Three conservation equations
  - → Rankine-Hugoniot equation
    - ✓ $E - E_0 = \frac{1}{2}(V_0 - V)(P + P_0)$ (4.8.2)
    - ✓ The relations among energy increase, volume change and shock pressure upon shock compression
    - ✓ This equation will be derived from the next slide.

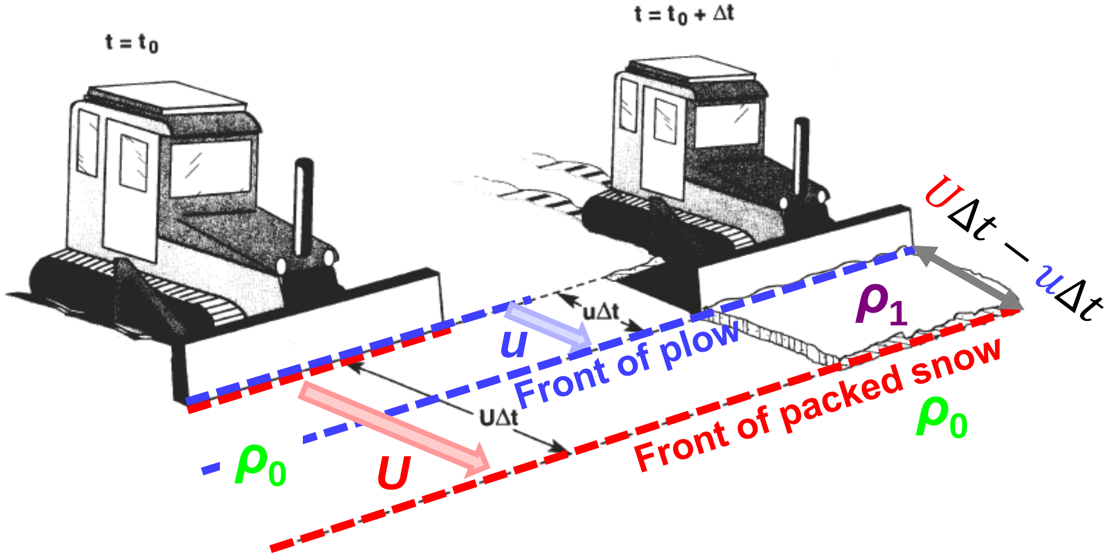
Conservation of mass -1

- At time t_0 the plow begins moving at velocity u from its starting point.
- At some later time, $t_0 + \Delta t$, the plow has moved a distance $u\Delta t$.
- The discontinuity between the loose and packed snow has moved a distance $U\Delta t$.
  - $U > u$
    - ✓ the discontinuity has advanced ahead of the plow



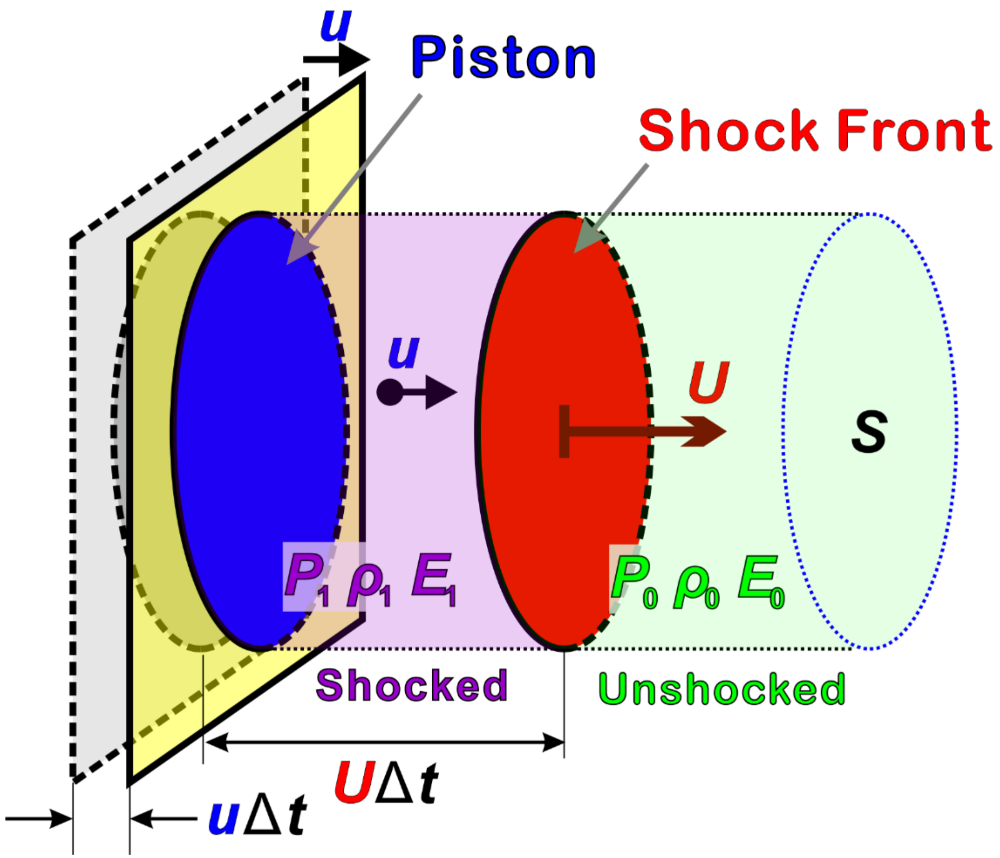
Conservation of mass -2

- ❑ T
  by
  - ➢
  - ➢
- ❑ T
  the
  - ➢
- ❑ E
  - ➢
  - ✓ Conservation of mass



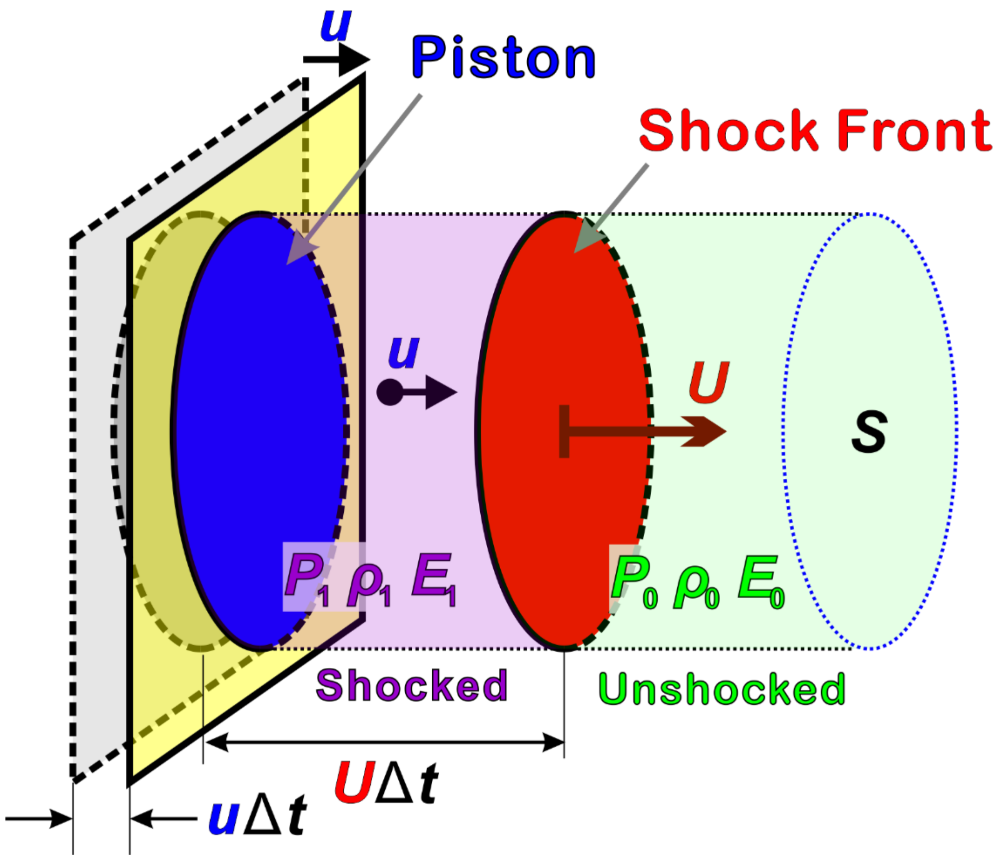
Piston – fluid system

- ❑ Piston-fluid system
  - ➢ Replacing the plow-snow system
- ❑ Reference frame: undisturbed fluid
  - ➢ Zero velocity
- ❑ Movement of piston at velocity u → shock wave with velocity U
  - ➢ Initial state "0" in front of the shock wave → state "1" behind the shock wave
- ❑ **Particle velocity**, u
  - ➢ The velocity of a particle of the fluid caught up in the flow
  - ➢ The shocked material carried with it
  - ➢ piston velocity



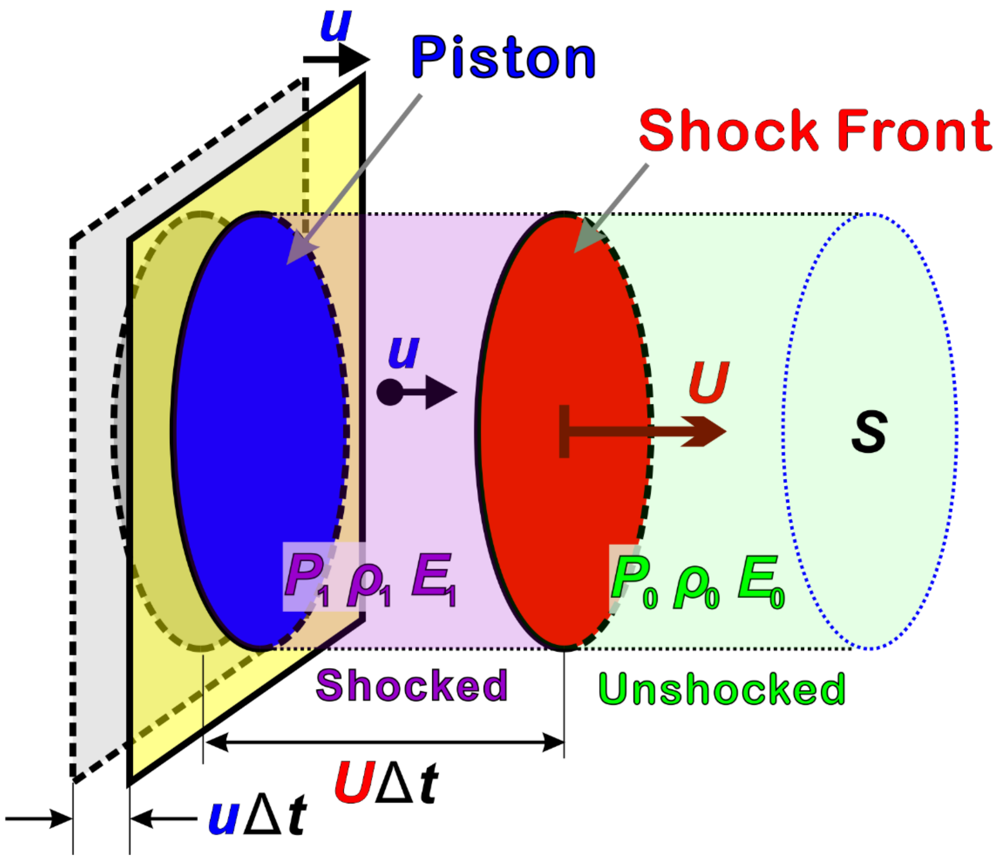
Conservation of momentum -1

- Pressures before and after the shock wave
  - P_0: the pressure in the unshocked part (before the shock)
    - ✓ from right to left
  - P_1: the pressure applied by the piston (after the shock)
    - ✓ from left to right
  - The net force on the system: $(P_1 - P_0)S$
  - The increase in momentum for Δt: $(P_1 - P_0)S\Delta t$



Conservation of momentum -2

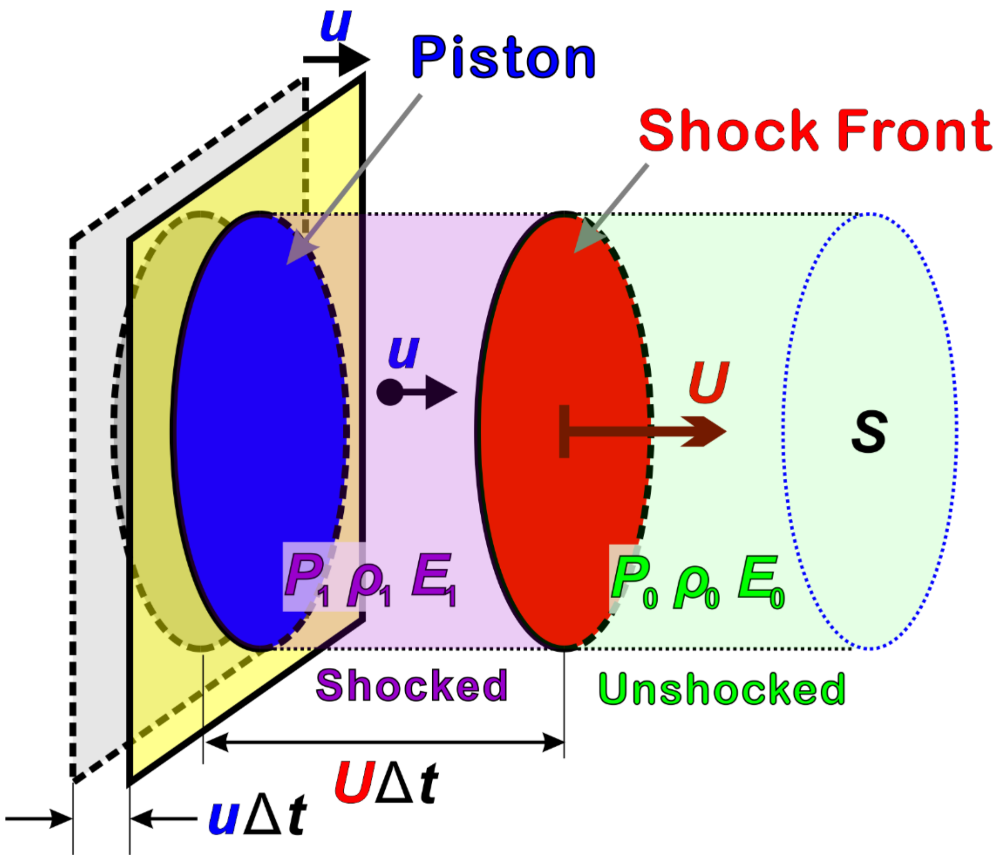
- Increase in momentum for Δt
  - The distance of the shock wave passing over: $U\Delta t$
  - Volume of the shock passing over: $U\Delta t \cdot S$
  - Mass of this volume: $\rho_0 \cdot U\Delta t S$
  - Increased momentum: $\rho_0 U\Delta t S \cdot u$
- Conservation of momentum
  - $(P_1 - P_0) S\Delta t = \rho_0 U\Delta t S u$
  - $P_1 - P_0 = \rho_0 U u \qquad (4.8.4)$



Conservation of energy -1

- The work done by the piston for Δt:
$W = Fx = P_1 S \cdot u\Delta t = P_1 uS\Delta t$
  - Force: $F = P_1 S$
  - Piston movement: $x = u\Delta t$

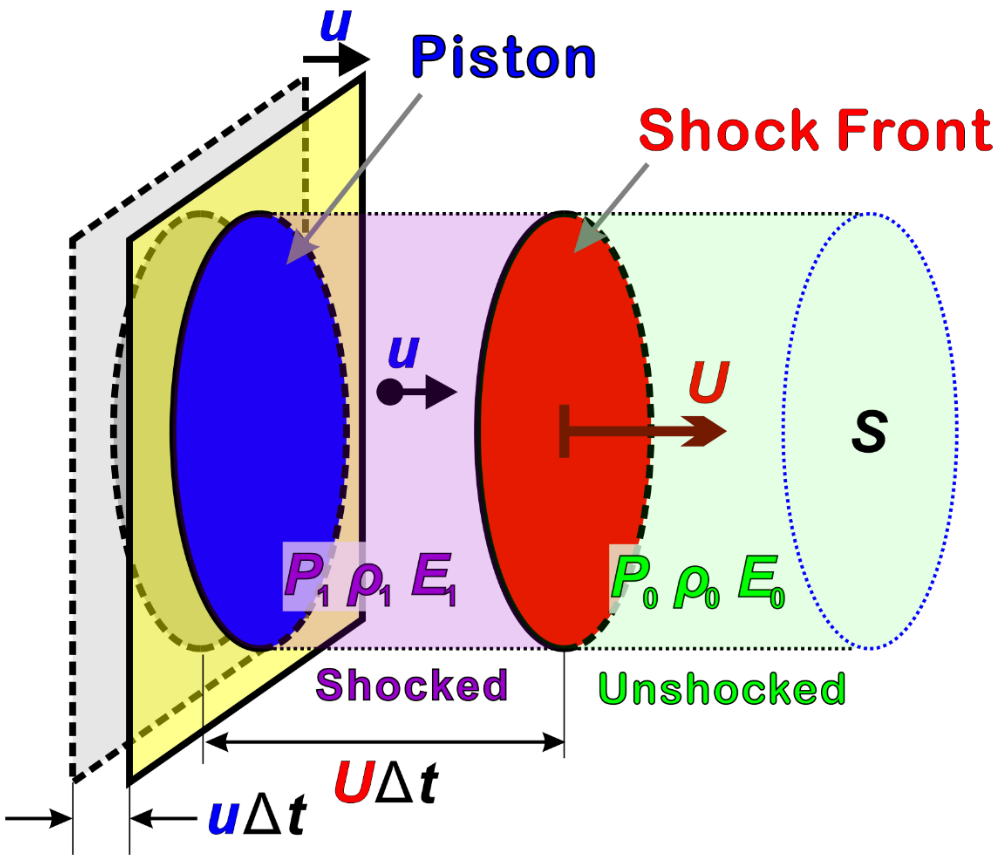
- The kinetic energy acquired by the volume where the shock wave has passed:
  - $\frac{1}{2}mu^2 = \frac{1}{2}(\rho_0 US\Delta t\,)u^2 = \frac{(\rho_0 Uu^2)S\Delta t}{2}$ (4.8.5)
    - ✓ The mass of material accelerated by the shock wave: $m = \rho_0 US\Delta t$



Conservation of energy -2

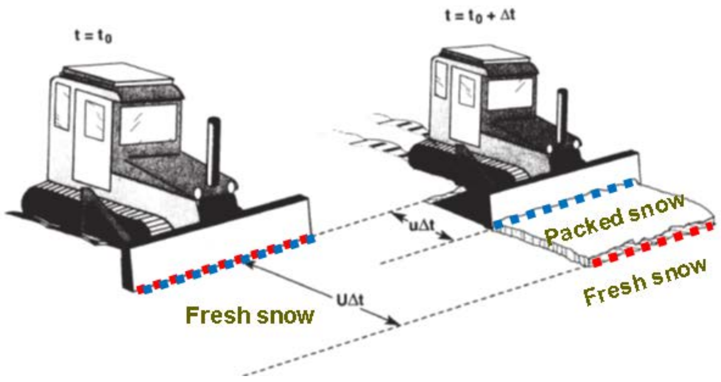
- The work done by the piston is also used to increase the internal energy in the volume of the shock wave passing over: $(E_1 - E_0)\rho_0 US\Delta t$
  - E_1, E_0: internal energy per mass
- The work done by the shock $P_1 u\Delta t$ becomes kinetic energy $\frac{(\rho_0 U u^2)S\Delta t}{2}$ and internal energy $(E_1 - E_0)\rho_0 US\Delta t$
  - $P_1 u = \frac{1}{2}\rho_0 U u^2 + \rho_0 U(E_1 - E_0)$

(4.8.6)



Rankine-Hugoniot equation -1

- The E change with respect to P and specific V ($V_1 \equiv 1/\rho_1$ and $V_0 \equiv 1/\rho_0$) from the initial to final states by eliminating the U and u from Eq. (4.8.3), (4.8.4) and (4.8.6)
  - (4.8.3): $\rho_0 U = \rho_1 (U - u) \rightarrow \rho_1 U - \rho_0 U = \rho_1 u$
    - ✓ $\rightarrow \left(\frac{1}{V_1} - \frac{1}{V_0}\right) U = \frac{1}{V_1} u \rightarrow \frac{V_0 - V_1}{V_0 V_1} U = \frac{1}{V_1} u$
    - ✓ $\rightarrow U = \frac{V_0}{V_0 - V_1} u$ (4.8.7)
      - If the volume becomes zero after the shock wave passing over ($V_1 = 0$), $U = u$
        - No packed snow in front of the snowplow.
      - If the volume does not decrease by the shock wave passing over ($V_1 = V_0$), $U = \infty$
        - The shock front propagates infinitely fast



If $\rho_0 = \rho_1$, the packed snow front is equal to the end of fresh snow.

Rankine-Hugoniot equation -2

- ❑ Conservation of momentum (4.8.4) $P_1 - P_0 = \rho_0 U u$ ← Conservation of mass (4.8.7) $U = \frac{V_0}{V_0 - V_1} u$
  - ➢ $P_1 - P_0 = \cancel{\rho_0}\left(\frac{\cancel{V_0}}{V_0 - V_1} u\right) u = \frac{u^2}{V_0 - V_1}$
  - ➢ → $u^2 = (P_1 - P_0)(V_0 - V_1)$ (4.8.8)
- ❑ Conservation of energy (4.8.6) $P_1 u = \frac{1}{2}\rho_0 U u^2 + \rho_0 U (E_1 - E_0)$ Uu ← Conservation of mass (4.8.7) $U = \frac{V_0}{V_0 - V_1} u$
  - ➢ $P_1 \cancel{u} = \frac{1}{2}\left(\cancel{\rho_0}\frac{\cancel{V_0}\cancel{u}}{V_0 - V_1} u^2\right) + \cancel{\rho_0}\frac{\cancel{V_0}\cancel{u}}{V_0 - V_1}(E_1 - E_0)$
  - ➢ → $P_1(V_0 - V_1) = \frac{1}{2} u^2 + (E_1 - E_0)$ (4.8.9)

Rankine-Hugoniot equation -3

- (4.8.9) $P_1(V_0 - V_1) = \frac{1}{2}u^2 + (E_1 - E_0)$ ← (4.8.8) $u^2 = (P_1 - P_0)(V_0 - V_1)$
  - $P_1(V_0 - V_1) = \frac{1}{2}[u^2] + (E_1 - E_0) = \frac{1}{2}[(P_1 - P_0)(V_0 - V_1)] + (E_1 - E_0)$
  - $E_1 - E_0 = P_1(V_0 - V_1) - \frac{1}{2}(P_1 - P_0)(V_0 - V_1)$
  - $E_1 - E_0 = \frac{1}{2}(V_0 - V_1)(P_1 + P_0)$ (4.8.2)
    - ✓ **Rankine-Hugoniot equation**
      - Jump conditions
      - The initial and final states achieved through the shock transition must be states of mechanical equilibrium

P, *V*, *E* at the shock compression

- Final volume
  - Conservation of mass (4.8.3) $\rho_0 U = \rho_1 (U - u)$
  - $\rightarrow \frac{\rho_0}{\rho_1} = \frac{V_1}{V_0} = \frac{U-u}{U} \rightarrow V_1 = V_0 \frac{U-u}{U}$ (4.8.3')
- Final pressure
  - Conservation of momentum (4.8.4) $P_1 - P_0 = \rho_0 U u$
  - $\rightarrow P_1 = P_0 + \rho_0 U u = P_0 + \frac{Uu}{V_0}$ (4.8.4')
- Final internal energy
  - Rankine-Hugoniot equation (4.8.2) $E_1 - E_0 = \frac{1}{2}(V_0 - V_1)(P_1 + P_0)$
  - $E_1 = E_0 + \frac{1}{2}\left(V_0 - V_0 \frac{U-u}{U}\right)\left(P_0 + \frac{Uu}{V_0} + P_0\right) = E_0 + V_0\left(1 - \frac{U-u}{U}\right)\left(P_0 + \frac{1}{2}\frac{Uu}{V_0}\right)$

(4.8.2')

T at the shock compression

- $E_1 - E_0 = \Delta E_{T_0} + \Delta E_{\mathrm{th}}$ (4.8.10)
  - ✓ ΔE_{T_0}: Internal energy increase by compression from V_0 to V_1 at T_0
  - ✓ ΔE_{th}: internal energy increase by heating from T_0 to T_1 at V_1

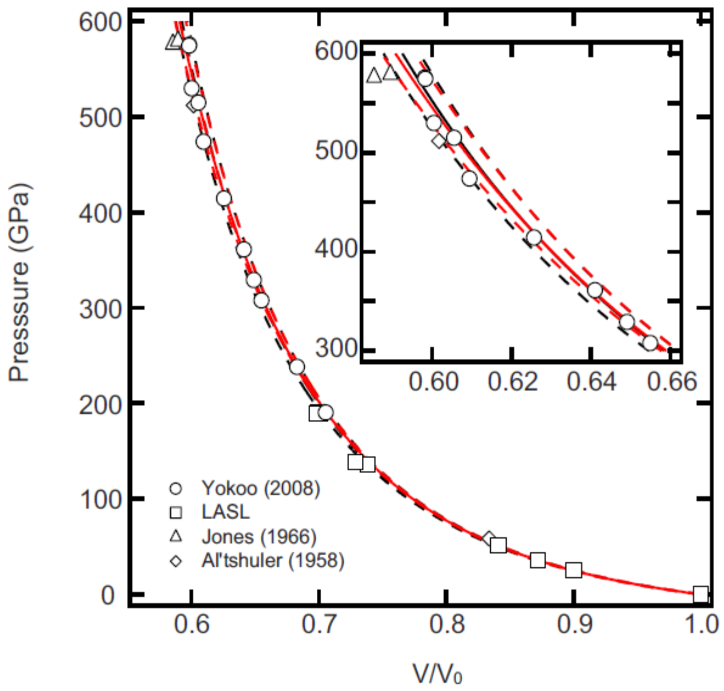
- $\Delta E_{\mathrm{th}} = \int_{T_0}^{T_1} C_V(V_1)\mathrm{d}T = 9nR\left(\frac{T}{\Theta_{\mathrm{D}}(V_1)}\right)^3 \int_{\frac{\Theta_{\mathrm{D}}(V_1)}{T_0}}^{\frac{\Theta_{\mathrm{D}}(V_1)}{T_1}} \frac{x^4 e^x}{(e^x-1)^2}\mathrm{d}T$ (4.8.11)
  - ➢ $\Theta_{\mathrm{D}}(V_1) = \Theta_{\mathrm{D}}(V_0)\exp\left[\frac{\gamma_0}{q}\left\{1-\left(\frac{V_1}{V_0}\right)^q\right\}\right]$ (4.8.12)

- The 1st and 2nd laws of thermodynamics (1.1.5) $\mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V$
  - ➢ $\Delta E_{T_0} = \int_{V_0}^{V_1}[T_0\mathrm{d}S - P_{T0}(V)\mathrm{d}V] = T_0[S(T_0,V_1) - S(T_0,V_0)] - \int_{V_0}^{V_1} P_{T0}(V)\mathrm{d}V$ (4.8.13)
    - ✓ $S(T_0,V_1) = \int_0^{T_0}\frac{C_V(V_1)}{T}\mathrm{d}T,\ S(T_0,V_0) = \int_0^{T_0}\frac{C_V(V_0)}{T}\mathrm{d}T$ (4.8.14)
    - ✓ $\int_{V_0}^{V_1} P_{T0}(V)\mathrm{d}V$: along the isothermal equation of state

- T_1 to is inversely calculated from Eq. (4.8.10) to (4.8.14)

Hugoniot

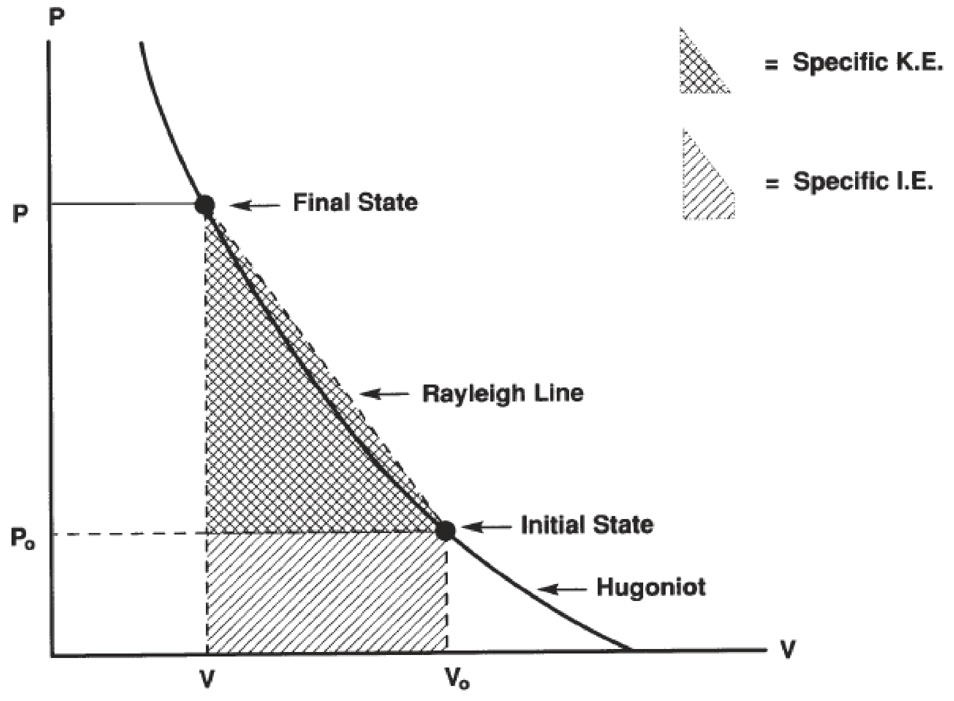
- Three equations (4.8.2, 3, 6) and one static EOS for five variables (P, V, E, U and u)
  - Only one independent variable
  - One curve in a P-V-E-U-u space
    - ✓ "**Rankine-Hugoniot curve**" or simply "**Hugoniot**"
    - ✓ Can be represented in any 2D plane (P-V, P-u, U-u,…)
    - ✓ Not a path that is followed during compression or any special thermodynamic path
    - ✓ The locus of all the possible end states that can be achieved behind a single shock wave passing through a material at a given initial state.



Gold (Yokoo et al., 2009)

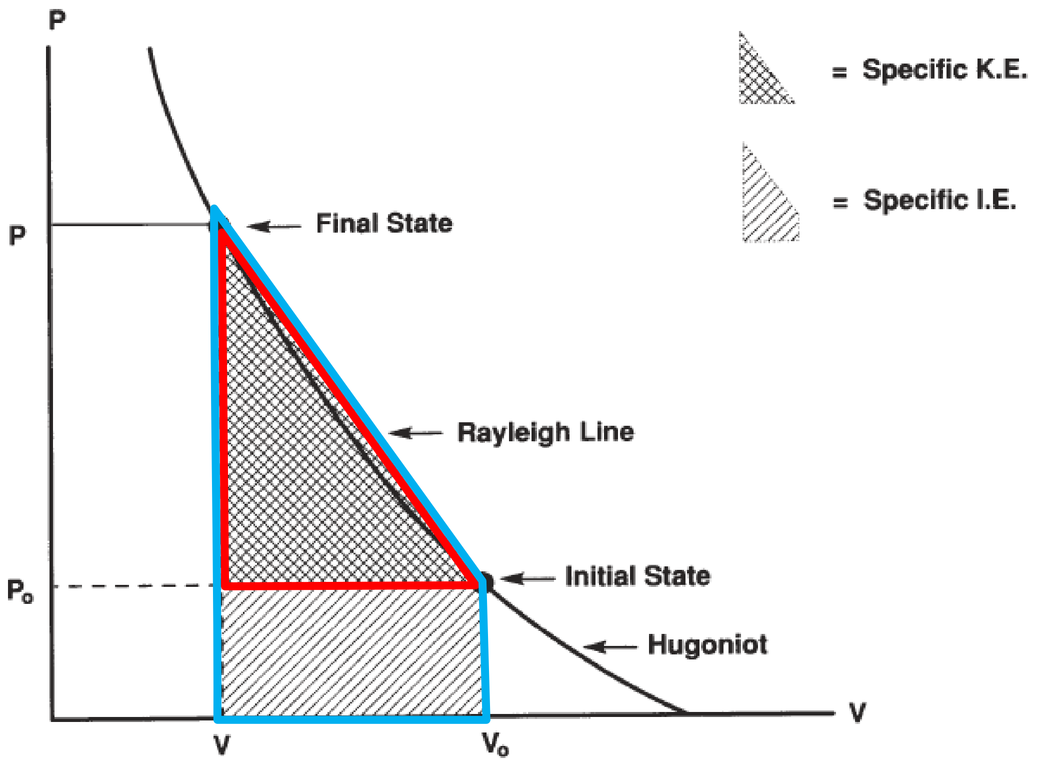
Rayleigh Line

- ❑ Rayleigh Line ($\mathcal{R}$): a line connecting the initial and final shocked state
- ❑ From (4.8.8) $u^2 = (P_1 - P_0)(V_0 - V_1)$ with (4.8.7) $U = \frac{V_0}{V_0 - V_1} u$
  - ➢ $u = \sqrt{(P_1 - P_0)(V_0 - V_1)}$
  - ➢ $U = \frac{V_0}{V_0 - V_1} u = \sqrt{\frac{P_1 - P_0}{V_0 - V_1}}$ (4.8.15)
- ❑ The slope of $\mathcal{R} = \frac{P_1 - P_0}{V_1 - V_0} = -\left(\frac{U}{V_0}\right)^2$ (4.8.16)
  - ➢ With increasing U, the slope becomes steeper rapidly.
    - ✓ Hugoniot: concave downward



Kinetic and internal energy increase by shock compression

- Kinetic energy from (4.8.8)
  - $\frac{1}{2}u^2 = \frac{1}{2}(P - P_0)(V_0 - V)$
- Internal energy increase from the Rankine-Hugoniot equation (4.8.2)
  - $E - E_1 = \frac{1}{2}(P + P_0)(V_0 - V)$
- If initially zero pressure ($P_0 = 0$),
  - $E - E_1 = \frac{1}{2}P(V_0 - V) = \frac{1}{2}u^2$ (4.8.11)
  - The internal energy increase is equal to the kinetic energy increase



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